Rencontre Poisson à La Rochelle and Poisson sigma model days

14 – 16 May 2025

Wednesday 14 May, afternoon, room MSI 100.

Chairleader: Pol Vanhaecke

14:30 – 15:20: Saïd Benayadi (Metz)

Algèbres Poisson-admissibles (Poisson admissible algebras)

La notion des algèbres de Poisson-admissibles nous permet de voir les algèbres de Poisson comme des algèbres non-associatives, avec un seul produit, qui sont Lie-admissibles avec des propriétés supplémentaires. De ce point de vue, il sera possible d'obtenir des informations intéressantes sur la structure des algèbres de Poisson et de déterminer des structures de Poisson sur certaines classes d'algèbres de Lie.

The notion of Poisson-admissible algebras allows us to see Poisson algebras as non-associative algebras, with a single product, which are Lie-admissible having additional properties. from this point of view, will make it possible to obtain interesting information about the structure of Poisson algebras and to determine Poisson structures on certain classes of Lie algebras.

15:30 – 16:20: Camille Laurent-Gengoux (Metz)

Classification of formal neighborhoods of a leaf of a singular foliation

We classify singular foliations admitting a given leaf and a given transverse singular foliation.
The talk is based on arXiv: 2401.05966.
Key words: Singular foliation, Lie group bundle, Yang-Mills connection.
This is a joint work with Simon-Raphael Fischer.

17:00 – 17:50: Gabriele Rembado (Montpellier)

Deformations and quantizations of moduli spaces of wild meromorphic connections

Fuchsian systems on the Riemann sphere have a natural Poisson space of parameters. Their admissible (= isomonodromic) deformations lead to a certain nonautonomous integrable Hamiltonian system, whose quantization is tantamount to the Knizhnik–Zamolodchikov connection in conformal field theory.
In this talk we will aim at a review of part of this story, and then present extensions involving linear systems of ODEs with irregular singularities. It is joint work with D. Calaque, G. Felder, R. Wentwort, M. Chaffe, L. Topley, D. Yamakawa, P. Boalch, J. Douçot, M. Tamiozzo.

Thursday 15 May, morning, room d'Orbigny C21.

Chairleader: Cyrille Ospel

9:00 – 9:50: Friedrich Wagemann (Nantes)

Lie cohomology of Lie-Rinehart algebras

This is joint work with Bas Jansens (TU Delft). Our objective in this work is a sort of "algebraic Gelfand-Fuchs cohomology", namely, we want to compute the Lie algebra cohomology of Lie-Rinehart algebras. Lie-Rinehart algebras are pairs (L,R) of a commutative associative algebra R and an R-module and k-Lie algebra L which are linked by an R-module and Lie algebra morphism (ancre) $\pi : L \to Der_k(R)$ (plus a compatibility condition). Lie-Rinehart algebras interpolate between the extreme cases $R = C^\infty(M)$ and $L = Vect(M)$ for a manifold M and R=k and L just a Lie algebra. The Lie algebra of sections of a Lie algebroid is an example, but a general Poisson algebra is not. We consider the order of differential operators and accordingly modules of order <= 1 for (L,R)-algebras. Our main results include the jet construction in this setting which transforms differential operators into R-linear operators, the proof that 1-cocycles are differential operators and the cohomology group H^1.

10:00 – 10:50: Antoine Caradot (Saint-Etienne)

(co)Poisson (co)algebras and vertex (co)algebra duality

In order to investigate the representation theory of a vertex algebra V, a fruitful strategy is to look at the properties of its C_2-algebra $R(V)$. This Poisson algebra reflects interesting properties of the vertex algebra and is often easier to handle than the vertex algebra itself. We are interested in studying vertex algebras in a closed monoidal category, and in providing a description of the dual versions of V and $R(V)$ when those exist. In this talk, we introduce vertex algebras graded by an abelian group and explain how to "dualize" the definition to obtain a graded vertex coalgebra. This leads to the notion of the C_2-coalgebra of a vertex coalgebra. We will describe its properties and show that the duality vertex algebra / vertex coalgebra passes down to a duality C_2-algebra / C_2-coalgebra. If time permits, we will also explain how these dualities carry on to the respective modules / comodules.

11:30 – 12:20: Andriy Panasyuk (Warsaw)

On a class of linear-quardratic Poisson pencils on $gl(N)$.

Joint work with Taras Skrypnyk.
Using two constant tensors c and b on $sl(N) \otimes sl(N)$ satisfying certain linear-quadratic relation, we build a quadratic Poisson structure on $gl(N)$ compatible with the standard linear Lie-Poisson structure. The case of N = 3 is considered in some details. The relation of the proposed brackets with the generalized classical Sklyanin algebra is also discussed.

Thursday 15 May, afternoon, room d'Orbigny C21.

Chairleader: Vladimir Salnikov

14:00 – 14:50: Pavol Ševera (Geneva)

Hopf algebras, Lie bialgebroids, and other beasts in the braided symplectic world.

Inspired by Chern-Simons theory, one can find symplectic analogues of quantum groups and their relatives in the symplectic category. These symplectic manifolds are moduli spaces of flat connections on surfaces with suitable boundary conditions, and the Lagrangian relations are given by 3d cobordisms. The picture becomes even more amusing once we consider braided Hopf algebras - symplectic manifolds get replaced by quasi-Hamiltonian spaces and the 3d cobordisms start floating in a soup.

15:00 – 15:50: Hadi Nahari (Paris)

The octonionic Hopf singular foliation: Minimal Lie groupoid and infinity algebroid

We investigate the leaf decomposition L of $\mathbb{R}^{16}$ induced by the Hopf construction for octonions. This leaf decomposition cannot be generated by any known Lie group action. In this talk, we describe the construction of a Lie groupoid G, whose orbits exactly match L. The associated Lie algebroid E is examined, highlighting its linear structure functions. The singular foliation induced by E is shown to be maximal among all modules with L as their leaf decomposition. We refer to it as the singular octonionic Hopf foliation.

Next, we extend E to a Lie 3-algebroid, enabling us to identify a representative for the universal Lie infinity algebroid of the singular octonionic Hopf foliation. This can be used to show that G and its associated Lie algebroid E have the minimal dimension within this framework.

A classical result on the octonionic Hopf leaf decomposition of $\mathbb{R}^{16}$ is that it cannot be generated by the orbits of a local isometric Lie group action. We extend this result by demonstrating that any singular foliation F with this leaf decomposition is not even Hausdorff Morita equivalent to a singular foliation induced by a local isometric action. This is joint work with Thomas Strobl.

16h30 – 17h20 : Thomas Strobl (Lyon)

(Very) informal historical overview of the Poisson sigma model and related subjects

Questions, maybe answers, followed or accompanied by a cocktail.

Friday 16 May, morning, room MSI 100.

Chairleader: Thomas Strobl

09:00 – 09:50: Rafał Suszek (Warsaw)

Of Fish and Men – A Reductionist's Toying with Principaloid Bundles

The classic gauge principle bases on the rigid symmetry model provided by a Lie group G acting as $\lambda : G \to \mathrm{Diff}(M)$ on a smooth manifold M, the latter representing the fibre of the configuration bundle of the physical system under consideration. The resultant gauge field theory employs principal G-bundles P and bundles associated with them through λ, and describes interactions of the Lie algebra-valued gauge field with charged matter fields with internal degrees of freedom given by M. The theory enjoys invariance under G-valued gauge transformations captured by the corresponding Ehresmann-Atiyah groupoid $\mathrm{At}(P)$. More fundamentally, the theory may be regarded as a model of dynamics on (the homotopy model of) the orbispace $M//G$. The descent $M \searrow M//G$ usually calls for an equivariantisation of geometric structures (*e.g.*, cohomology classes or gerbes) populating M, as demonstrated amply in the formulation of the gauge principle for the 2d σ-model in nontrivial target topology by Gawędzki, Waldorf and myself.

In my talk, I shall present a proposal, formulated jointly with Thomas Strobl, for a gauge principle based on a generalised symmetry model, given by an arbitrary Lie groupoid $\mathbf{G}$ with object manifold M. Such a generalisation is expected to elucidate the nature of Lie-algebroidal gauge symmetries previously observed by Thomas *et al* in, *i.a.*, the celebrated Poisson σ-model. In the new approach, the basic construct is a **principaloid bundle P** – a fibre bundle with typical fibre $\mathbf{G}$ and structure group given by the group $\mathbf{B}$ of bisections of $\mathbf{G}$. The principaloid bundle canonically determines a configurational field bundle with typical fibre M, acted upon by its Ehresmann-Atiyah groupoid $\mathrm{At}(\mathbf{P})$. I shall discuss essential novel features of this gauge field-theoretic construction and propose simple models of field dynamics invariant under $\mathbf{B}$-valued gauge transformation. This inevitably leads me to the notion of (further) reduction of the structure group depending on the geometric structure present on M. A possible reduction scheme—using Cartan's association—shall be indicated. Time permitting, I shall also touch upon the subtle issue of cohomological descent of field theories with configuration fibre M defined in terms of Cheeger-Simons differential characters, such as the 2d σ-models of loop mechanics, to the orbispace $M//\mathbf{G}$, in which the Bott-Schulman-Stasheff cohomology of the structure groupoid shall be seen to play an essential role. This shall bring me, *via* Xu's work, back to the symplectic category, and that consistently with the higher-geometric approach to σ-models worked out jointly with Ingo Runkel.

10h00 – 10h50: Olga Chekeres (L'Aquila)

Odd and generalized Wilson surfaces

In this talk I discuss various extensions and generalizations of Wilson surface observables in gauge theories. Previously, Wilson surface observables were interpreted as a class of Poisson sigma models. We profit from this construction to define and study the super version of Wilson surfaces. We provide some 'proof of concept' examples to illustrate modifications resulting from appearance of odd degrees of freedom in the target. We also explain some natural directions for defining the analogues of Wilson surface observables in higher dimensions. The talk is mostly based on 'Odd Wilson surfaces', J. of Geometry and Physics 203, 2024 (arXiv:2403.09820). This is a joint work with V. Salnikov. Time permitting, I will talk about a new construction of weird Lie superalgebras, discuss their basic properties and certain classes of examples. This latter part of the talk is based on a work in progress with A. Kotov and V. Salnikov.

11:30 – 12:20: Alexei Kotov (Hradec Králové)

DG bundles in geometry and mathematical physics

A DG manifold is a Z-graded supermanifold endowed with a homological vector field of degree 1. DG manifolds form a category whose morphisms are degree-preserving maps of such that the corresponding vector fields are f-related. In this talk I consider fibered bundles in the category of DG manifolds. Drawing primarily on my joint work with Thomas Strobl, I will show how DG bundles arise in the description of sigma models in mathematical physics and characteristic classes in geometry.